PreCalculus Flexible Instructional Days
Parametric Equations

Name: ___________________

"A day without math is like a day without sunshine."
"A day with snow is a day with math!"
(snow day version)

One of the topics essential to the PreCalculus curriculum at Red Lion is Parametric Equations. We will use the ***PA Flexible Instructional Days*** as our opportunity to instruct on and learn about parametric equations. This packet has been carefully designed to take the place of daily instruction with each lesson lasting approximately 45 minutes. Any lessons not completed during the Flexible Instructional Day must be completed within 2 weeks for appropriate credit and attendance.

Please note each of the methods of instruction and learning that will be used during this packet. Do not skip over any sections in order to just "hurry up and finish the assignment." Use email **naylorj@rlasd.net** for questions during a flexible day.

Method 1 – Notes
Follow along by reading and filling in blanks as appropriate.

Method 2 – Video
Watch the corresponding video to see / hear the math developed

All videos available by QR Code or by visiting Flexible Instructional Days located on **mrnaylorswebplace.com**

Method 3 – Practice
Give it a try and see if you understand (before starting the assignment)

Solutions available on last page

Method 4 – Assignment!
These problems will be graded for correctness (on a separate sheet of paper) to be turned in on the assigned date. (TBA)

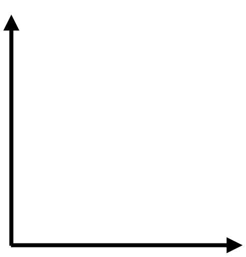
Begin Flexible Day #1

Using a popular web search engine, (or a dictionary) obtain a simple but accurate definition for ***Parametric Equations.***

Definition - ____

____

Suppose the equation $y = -\frac{x^2}{72} + x$ represents the path of a projected objected with an initial velocity of 48 ft/sec. "y" represents the vertical height and "x" represents the horizontal position. Graph this equation on a calculator using the **WINDOW 0≤x≤80 0≤y≤30.**



- ✓ Notice (by tracing) that the max height of 18 feet occurs when the object's horizontal distance is 36 ft
- ✓ Notice (by tracing) that the final horizontal distance is x=72 ft when the object returns to the ground
- ✓ Label these points on your sketch

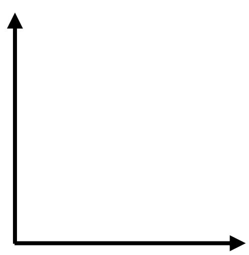
Although this equation is helpful in modeling the <u>position</u> of the projected object, it is ***not able*** to model the ***time*** when the object will reach these heights. To remedy this *time dilemma* we introduce a 3rd variable "t" called a parameter.

$$x = 24\sqrt{2}t$$

$$y = -16t^2 + 24\sqrt{2}t$$

square root of just 2 not square root of 2t

The given parametric equations allow the "x" and "y" positions to be written as functions of time. Do not worry about magically creating these equations… we will create them later. Instead, let's graph these parametric equations by changing the **calculator mode** from **Func → Par.** Use the same xy window, but make sure the Tmin=0, Tmax=4 and Tstep=0.1 (T represents seconds, so this ensures 4 secs of graphing with a point plotted every tenth of a second.)



- ✓ Notice (by tracing) that when the time =1.06 seconds, we reach the maximum.
- ✓ Notice (by tracing) that when the time = 2.12 seconds the object returns to the ground
- ✓ Use the trace button to investigate these points and label them on your sketch.

The power of parametric equations is to ***<u>match</u> times*** with ***positions***. In fact, we could find the position of the object at <u>any</u> time by simply plugging a "t" value into the equations.

For example, at 1 second, the horizontal and vertical positions would be given by…

$$x = 24\sqrt{2}(1) = 33.94 \ feet$$

$$y = -16(1)^2 + 24\sqrt{2}(1) = 17.94 \ feet$$

You find the horizontal and vertical positions at 2 seconds

$x =$

$y =$

Day 1 continues onto next page →

The original equation we used, $y=-\frac{x^2}{72}+x$, is called a rectangular equation. Label it as such back on page 1. Rectangular uses just x & y. But when the parameter "t" is used, we have a set of parametric equations. Also label this set of equations back on page 1.

So how do we know for sure that these two different types of equations, that modeled projectile motion, are actually equal (besides the fact that the graphs look the same?) It is actually simple to tell using a process known as "eliminating the parameter." Watch this short video, also available on **mrnaylorswebplace.com.** Be sure to follow along, using this blank space to show the work.

On our next Flexible Day, you will learn how to ***analyze*** parametric equations with a labeled graph and an equivalent converted equation. But before moving on, take a moment to make a simple statement, noting one new idea learned from Day One's Introductory Lesson…

New Idea: ____

____

End Flexible Day #1

Begin Flexible Day #2

In order to correctly and fully analyze parametric equations, we will utilize(1) an appropriate table of values, (2) label the initial and terminal point and (3) show the orientation (direction) of the graph. We can also (4) convert to rectangular by eliminating the parameter. Use the following short video to watch how to analyze the following parametric equation. Be sure to follow / write as the video goes along…

$$x=t^2-4 \qquad -2\le t\le 3$$
$$y=\frac{t}{2}$$

Now it is your turn to analyze. Remember, make a table that corresponds to the parameter. Create a sketch, showing the orientation (direction) and label the initial and terminal points. Then eliminate the parameter (convert) to a rectangular equation, creating an appropriate ***domain restriction*** with the final answer.

$$x=\sqrt{t}$$
$$y=2+t$$

Converting parametric to rectangular is called "eliminating the parameter," as discussed in the previous examples. But what if we want to convert the other direction… rectangular to parametric? The typical confusion with students is the fact that there can be numerous rectangular equations that would be equivalent to a single parametric equation. Let us look at this more closely…

Suppose $y=5-7x$ needs to be converted to parametric. Suppose also that you are told that $x=2t-4$. Now a simple substitution will allow the original y= equation to become parametric…

$$y=5-7(2t-4)$$
$$y=5-14t+28$$
$$y=\_\_\_\_\_\_\_\_\_\_ \text{ (you finish)}$$

So the parametric answer would require ***both*** the y= and x= equations for a final parametric of →

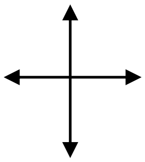
$$x=2t-4$$
$$y=33-14t$$

So what if you had been told that $x=3t+1$? Well, you would still substitute, but the y= equation would of course be different. GO ahead, and find these parametric equations.

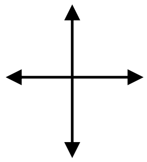
Day 2 continues onto next page →

With all of this converting, don't forget that the rectangular and parametric equations should still be producing the same graphs… after all, they are equivalent. Using your calculator, graph the original $y = 5 - 7x$ (Function Mode). Then graph the parametric equations you created. (Parametric Mode)

Rectangular



Parametric



Look the Same?

They Should!

Before moving on, take a moment to make a simple statement, noting one new idea learned from Day Two's Analysis of Parametrics Lesson…

New Idea: ______________________________________

____

End Flexible Day #2

Begin Flexible Day #3

Remember to do this assignment for correctness. Remember, it is also be counted towards your attendance, so PLEASE complete it.

p. 673 1 - 6 Match with or without a calculator… you decide.
p. 673 7,8 Be sure to do each step. #8 – calc in radians!
p. 673 10 Do all steps and use a parameter restriction of $0 \le t \le 6$
p. 673 11 Do all steps and use a parameter restriction of $0 \le t \le 4$
p. 674 15 Do all steps. Be careful…the rectangular answer needs a domain restriction to restrict the parabola.
p. 674 43 Let x = 2t for 1st set… then make up your own x= ? for 2nd
p. 674 48 Let x = t – 7 for 1st set… then make up your own x= ? for 2nd
Also, graph your 1st equations to PROVE that they are in fact the same

p. 675 62 You must give / show a reason / work for your answer.

End Flexible Day #3

Begin Flexible Day #4

As we discussed on Day #1, parametric equations can be used to model projectile motion. The official English Units parametric projectile equations incorporate the initial height (h) measured in feet, the initial angle (θ), and the initial velocity (v_o) measured in feet per second. The -16 is the acceleration due to gravity, measured in feet per sec^2.

$$x = (v_o \cos\theta)t$$

$$y = h + (v_o \sin\theta)t - 16t^2$$

Suppose a bungee launcher, 5 feet above the ground, sends a water balloon across a field with an initial velocity of 50 feet/sec at a 40° angle.

→ Set up the horizontal and vertical projectile equations

→ Use the parametric projectile formulas to find the horizontal and vertical distances after 2 seconds.

→ Could you find the time until the balloon hits the ground? Go ahead and try! Show / explain any work.

Let's use p.675 #59 to further investigate projectile motion. We will use a graphing method for (b), then an algebra method for (c). (d) is a real treat ☺. Be sure to have your calculator handy and follow along on this paper as you watch the video.

(a) (b)

(c) (d)

Day 4 continues onto next page →

Exit Ticket

$x = (v_0 \cos\theta)t \quad y = h + (v_0 \sin\theta)t - 4.9t^2$ where distance is in <u>meters</u> and time is in seconds

An archer shoots an arrow with an initial velocity of 65 m/s at an angle of 4.5° with the horizontal at a target 70 meters away. The archer holds the bow 1.5 meters above the ground when she shoots the arrow. If the bull's eye is exactly 1.5 meters high, will she make a direct hit? By how much does she miss vertically (if at all)?

You must use an Algebra Method on the Front and a Graphing Method on the Back

Algebra Hint: … you will want to start by finding the <u>time</u> it takes to horizontally traverse 70 meters, then…

Find the Projectile Exit Ticket from p.3. Carefully detach and complete as it will be turned in as part of your Flexible Day #4 credits and attendance.

End Flexible Day #4

Begin Flexible Day #5

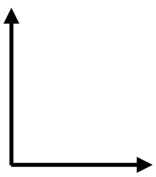
As a final activity involving parametric equations and projectile motion, complete the Golf Club Problem. This completed assignment will be counted for your Day #5 attendance and graded for correctness.

End Flexible Day #5

Solutions and answer key for practice sessions throughout packet

Flexible Day #1

Quick Sketch of Graph



Horizontal and Vertical Positions after 2 seconds…

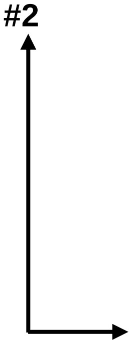
$$x = 24\sqrt{2}(2) = 67.88\ feet$$

$$y = -16(2)^2 + 24\sqrt{2}(2) = 3.88\ feet$$

Flexible Day #2

Suggested table based on equations

t	x	y
0	0	2
4	2	6
9	3	11
16	4	18
25	5	27



$x = \sqrt{t}$ so if you square both sides $x^2 = t$

Since $y = 2 + t$ substitution gives $y = 2 + x^2$

Nicely written → $y = x^2 + 2$

But this is the equation of a FULL PARABOLA.
So we correctly create a domain restriction in order to only graph the right half of the parabola…

$y = x^2 + 2,\ x \geq 0$

Flexible Day #2 continued

$$y = 5 - 7(3t + 1)$$
$$y = 5 - 21t - 7$$
$$y = -21t - 2$$

So final parametric answer → $y = -21t - 2$, $x = 3t + 1$

Flexible Day #4

Just plug 2 in for "t" after correctly setting up the equations with the initial numbers

$$x = (50\cos 40)2$$
$$= \ 38.30.. \times 2 = 76.60\ ft$$
$$y = 5 + (50\sin 40)2 - 16(2)^2$$
$$= 5 + \ 32.13... \times 2 - 64 = 5.28\ ft$$

Time to hit the ground…. Hmmm…., the ground would be when y = _______ It is a bit tricky to solve that equation for "t" when it equals zero… but you can try. If that is too hard, just graph the parametrics (you create the window) and TRACE to the special place. (No answer given… you try to find the value!)

Flexible Day #5

This assignment ends up being a lot of calculator computing, but when you are all done, you will be able to compare all of the numbers and be the best caddy for Joe! Done shoveling all the snow? Try the bonus problem

If we do not have all 5 flexible days this year, in class time will be given to finish this packet. A quiz will likely follow, so be sure you understand the math, not just completing the problems.

Any further questions can be directed to Mr. Naylor via email (naylorj@rlasd.net) on a Flexible day, or asked in class on a school day.

Exit Ticket

Your graphing calculator support should be a sketch showing the bull's eye's location with at least one important <u>point</u> <u>marked</u> that supports your final answer.

Final Answer: Hit the Bull's Eye? **-YES-** **-NO-**

Missed vertically by ___________ meters (if applicable)